1 Fitness derivations

Let $W_{ijk}$ be the fitness of an individual with behavior $i$, marker $j$, in subpopulation $k$. Using the assumptions in the text, the expression is:

$$W_{ijk} = e(p_{ik}(1 + \delta) + (1 - p_{ik})(1)) + (1 - e)\left(\frac{x_{ijk}}{q_{jk}}(1 + \delta) + \frac{q_{jk} - x_{ijk}}{q_{jk}}(1)\right)$$

$$= e(1 + \delta p_{ik}) + (1 - e)\left(1 + \delta \frac{x_{ijk}}{q_{jk}}\right).$$

Collecting the $\delta$ terms,

$$W_{ijk} = 1 + \delta \left( e p_{ik} + (1 - e) \frac{x_{ijk}}{q_{jk}} \right).$$

Using the definition of the covariance, $D$, between $i$ and $j$, we get:

$$W_{11k} = 1 + \delta \left( e p_{1k} + (1 - e) \frac{p_{1k}q_{1k} + D_{k}}{q_{1k}} \right),$$

$$W_{00k} = 1 + \delta \left( e p_{0k} + (1 - e) \frac{p_{0k}q_{0k} + D_{k}}{q_{0k}} \right),$$

$$W_{10k} = 1 + \delta \left( e p_{1k} + (1 - e) \frac{p_{1k}q_{0k} - D_{k}}{q_{0k}} \right),$$

$$W_{01k} = 1 + \delta \left( e p_{0k} + (1 - e) \frac{p_{0k}q_{1k} - D_{k}}{q_{1k}} \right).$$

We use these to simplify expression (1) in the text. Although we do not mention it in the text, $D$ is formally identical to the population genetics concept of linkage disequilibrium, which is just the covariance between alleles at two loci.
2 Derivation of $\Delta p_{ik}$

By definition $\Delta p_{ik} = \Delta x_{11k} + \Delta x_{10k}$. Assuming the replicator dynamic for transmission:

$$\Delta p_{ik} = x_{11k}(W_{11k} - \overline{W}_k) + x_{10k}(W_{10k} - \overline{W}_k).$$

This simplifies to:

$$\Delta p_{ik} = (1 - p_{ik})(x_{11k}W_{11k} + x_{10k}W_{10k}) - p_{ik}(x_{01k}W_{01k} + x_{00k}W_{00k}).$$

Substituting in fitness expressions in section 1 and using the definitions of $x_{ijk}$ in terms of $p, q, D$, the above simplifies to:

$$\Delta p_{ik} = \delta \left( e p_{1k} p_{0k} (p_{1k} - p_{0k}) + \frac{1 - e}{q_{1k} q_{0k}} (p_{1k} q_{0k} q_{1k} - D^2_k)(p_{1k} - p_{0k}) \right).$$

Factoring and simplifying further:

$$\Delta p_{ik} = \delta p_{1k} p_{0k} (p_{1k} - p_{0k}) \left( 1 - (1 - e) \frac{D^2_k}{p_{1k} p_{0k} q_{1k} q_{0k}} \right).$$

The above is the expression (2) in the text.

3 Derivation of $\Delta q_{jk}$

As with $\Delta p_{ik}$:

$$\Delta q_{1k} = \Delta x_{11k} + \Delta x_{01k}$$

$$= q_{0k}(x_{11k}W_{11k} + x_{01k}W_{01k}) - q_{1k}(x_{10k}W_{10k} + x_{00k}W_{00k}).$$

Again, we make substitutions for fitness expressions and convert all $x_{ijk}$ to be in terms of $p, q, D$. Simplifying:

$$\Delta q_{1k} = 2\delta D_k (p_{1k} - p_{0k}) \left( 1 - \frac{e}{2} - (1 - e) \frac{D_k}{q_{1k} q_{0k}} \left( \frac{q_{1k} - q_{0k}}{p_{1k} - p_{0k}} \right) \right).$$

The above is expression (3) in the text.
4 Derivation of $\Delta D_k$

By definition, $\Delta D_k = D'_k - D_k$. Expanded, this is:

$$\Delta D_k = x_{11k} \frac{W_{11k}}{W_k} x_{00k} \frac{W_{00k}}{W_k} - x_{10k} \frac{W_{10k}}{W_k} x_{01k} \frac{W_{01k}}{W_k} - D_k.$$ 

Substituting fitness expressions and expressing all $x_{ijk}$ in terms of $p, q, D$, then assuming all terms of order $D^3_k \approx 0$, selection is weak such that $W_k \approx 1$ and $\delta^2 \approx 0$ yields:

$$\Delta D_k = \delta \left( D_k (2p_{1k}p_{0k}(1-e) + 1) + (1-e) \frac{D_k^2}{q_{1k}q_{0k}} (p_{1k}p_{0k})(q_{1k}q_{0k}) \right).$$

Assuming $D_k$ is small enough that $D_k^2 \approx 0$ gives expression (4) in the text.